Philosophy of Logic

On frameworks for doing logic

João Marcos

UFSC

2024.1

Logic as a mother-structure?

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Algebraic structures, ordered structures, topological structures, and...
... logical structures!

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Consider $\langle \mathcal{S}, \rhd \rangle$, where:

- $\mathcal{S}$ is a collection of sentences

What are sentences?

- anything we want?
- same as *formulas*?
- same as *propositions*?
- should we impose some relevant structure?

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... logical structures!

Consider $\langle \mathcal{S}, \rhd \rangle$, where:

- $\mathcal{S}$ is a collection of sentences
- $\rhd$ is a collection of consecutions

What type do the consecutions have?

- do they represent an operator, or a relation?
- a unary relation? a binary one? something else?
- a single premiss? many premisses?
- a single conclusion? multiple alternatives?
- are the sentences organized into: sets? bags? sequences?

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Some popular frameworks for doing logic:

- Set $\longrightarrow$ Set(Set): closure operators

Opportunities:
— a built-in notion of logical theory
...

Challenges:
— displaying the non-determinism of inference
— axiomatizing the 'right' notion of consequence
...

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- Fmla: collections of theses

Opportunities:

— implements a notion of assertion

...

Challenges:

— there are 'conclusions', but no premisses!

— no built-in notion of following-from

— why not look at antitheses, as well?

— no intuitive logical interpretation

— hard to distinguish one logic from another

...

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- Set $\longrightarrow$ Set(Set): closure operators
- Fmla: collections of theses
- Set-Fmla: single-conclusion consequence relations (cr)

Opportunities:

— captures the givens and the goal
— connection to proof-from-premisses
— connection to truth-preservation
...

Challenges:

— what about Fmla-Set?
— why privilege truth over falsity?
— hard to capture the meaning of certain constants
— still hard to distinguish one logic from another
...

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- Fmla: collections of theses
- Set-Fmla: single-conclusion consequence relations (cr)
- Set-Set: multiple-conclusion / generalized consequence relations (gcr)

Opportunities:

— implements also a notion of denial
— demands less from the object language
— neat characterization of constants
— geometrical view on proofs
— logics are reconciled with their models
— abstractly capturing logical principles
— restores perfect symmetry
— generalizes all the previous cases
...

Challenges:

— interpreting what's going on?
...

Time for judgment!

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- compatibility, S-consequence, T-consequence, T-closure, S-closure

(see slide 1 of “RecaptureA” and slides 5–7 of “MCL-Tut-1”)

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- gcrs and logical principles

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- gcrs and logical constants as punctuation marks

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- gcrs and logical constants as punctuation marks

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- gcrs and a theory of opposition

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