Philosophy of Logic

Some fundamental logical properties

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What makes for a good logic?

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Some desirable properties

- substitution-invariance
- finitariness
- congruentiality
- extensionality
- truth-functionality
- soundness & completeness

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Combining logics

- how can two logics be merged into one?
- what happens from a deductive / semantical perspective?

Substitution-invariance

a.k.a. ‘structurality’

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"Logics are formal"

The birth of 'abstract logics':

- on the algebraic structure of sentences
- what is a uniform substitution?
- on consequence-preserving substitutions:

$$\Pi \rhd \Sigma \Longrightarrow \Pi^{\varepsilon} \rhd \Sigma^{\varepsilon}$$

for every endomorphism ε on the algebra of sentences

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How formal can a nonmonotonic logic be?

- the bridges that fall...
- relettering allowed!

Finitariness

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"No role for the infinite in reasoning"

- compactness and choice
- the deductive viewpoint: on the role of ω-rules
- the semantical viewpoint: on the finitary character of inconsistency
- the abstract viewpoint: compatibility, consequence, and closure

Congruentiality

a.k.a. ‘self-extensionality’

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"Equivalent sentences are indistinguishable"

- on the concept of logical equivalence, $\lhd\!\rhd$, and on demanding the logical constants to be compatible with the latter notion
- an abstract definition (1-ary case):

$$\varphi \lhd\!\rhd \psi \Longrightarrow ©\varphi \lhd\!\rhd ©\psi$$

  for every connective ©
- the algebraic outlook: taking the quotient of a structure
- on the connection to modalities

Extensionality

Extensionality

"Judgments are all that matters"

- not distinguishing between two asserted sentences
- not distinguishing between two denied sentences
- on the connections to positive / negative modalities
- abstract definitions (1-ary case):

$$\varphi, \psi, ©\varphi \triangleright ©\psi$$

$$©\varphi \triangleright ©\psi, \varphi, \psi$$

Truth-functionality

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“All that matters is the value one gives to things”

- compositionality in its purest form
- from a deductive viewpoint: a form of analyticity
- on the abstract counterpart:
  capturing characterizability by a single logical matrix
- important to emphasize: a property of logic, not just of its semantics!

Soundness & Completeness

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Comparing two logics to one another

Given logics $\langle \mathbf{Fm}, \rhd_1 \rangle$ and $\langle \mathbf{Fm}, \rhd_2 \rangle$, we say that:

- $\langle \mathbf{Fm}, \rhd_1 \rangle$ is *sound* with respect to $\langle \mathbf{Fm}, \rhd_2 \rangle$ if $\rhd_1 \subseteq \rhd_2$
- $\langle \mathbf{Fm}, \rhd_1 \rangle$ is *complete* with respect to $\langle \mathbf{Fm}, \rhd_2 \rangle$ if $\rhd_1 \supseteq \rhd_2$

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Translating logics into one another

Given logics $\mathcal{L}_1 := \langle \mathbf{Fm}_1, \rhd_1 \rangle$ and $\mathcal{L}_2 := \langle \mathbf{Fm}_2, \rhd_2 \rangle$, we say that:

- a translation of $\mathcal{L}_1$ into $\mathcal{L}_2$ is a mapping $\star : \mathbf{Fm}_1 \longrightarrow \mathbf{Fm}_2$ such that

$$\Pi \rhd_1 \Sigma \Longrightarrow \Pi^\star \rhd_2 \Sigma^\star \qquad \text{(preservation)}$$

- a translation of $\mathcal{L}_1$ into $\mathcal{L}_2$ is said to be conservative if

$$\Pi \rhd_1 \Sigma \Longleftarrow \Pi^\star \rhd_2 \Sigma^\star \qquad \text{(reflection)}$$

Combining logics

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Let's reason together!

- the different mechanisms out there: fusion, product, fibring, etc
- the abstract problem of fibring logics:
  producing the least common conservative extension of two given logics
- the problem of shared vocabulary
- the easy problem of finding a deductive counterpart to fibring
- the tough problem of finding a semantical counterpart to fibring