

Philosophy of Logic

From a structural viewpoint

João Marcos

UFSC

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An (updated) Architecture of Mathematics

On Mathematic
sry

[MacLane, 1986]

Activity	Idea	Formulation
Collecting	Collection	Set (of elements)
Counting	Next	Successor; order Ordinal number
Comparing	Enumeration	Bijection Cardinal number
Computing	Combination (of nos)	Rules for addition Rules for multiplication Abelian group
Rearranging	Permutation	Bijection Permutation group
Timing	Before and after	Linear order
Observing	Symmetry	Transformation group
Building, shaping	Figure; symmetry	Collection of points
Measuring	Distance; extent	Metric space
Moving	Change	Rigid motion Transformation group Rate of change
Estimating	Approximation	Continuity Limit
	Nearby	Topological space
Selecting	Part	Subset Boolean algebra
Arguing	Proof	Logical connectives
Choosing	Chance	Probability (favorable/total)
Successive actions	Followed by	Composition Transformation group

An (updated) Architecture of Mathematics

The great types of mathematical structures

[Bourbaki, 1950]

- mother-structures
 - ▶ algebras
 - ▶ orderings
 - ▶ topologies
 - ▶ ...
- multiple structures
 - ▶ topological algebra?
 - ▶ algebraic topology?
 - ▶ ...
 - ▶ logic!

What is a logical structure?

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Such stuff as logics are made on

- formulas
- inferences
- (arguments?)
- proofs
- models
- ???

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The role of syntax

Capturing *purely structural features* of:

- sentences
- consecutions

A matter of attitude?

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What are logical consecutions about?

- expressing 'sequents'
- making 'judgments'
(recall Frege's *Inhaltsstrich* & *Urteilsstrich*)
- rules for type formation, term introduction / elimination, computation

Das Urtheil.

§ 2. Ein Urtheil werde immer mit Hilfe des Zeichens



ausgedrückt, welches links von dem Zeichen oder der Zeichen-
verbindung steht, die den Inhalt des Urtheils angiebt.

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TO DISCUSS: What shape should a consecution take?

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TO DISCUSS: What shape should a consecution take?

What does one use consecutions for?

- identifying logical principles / laws
- expressing 'valid' patterns of reasoning
- providing meaning to the logical constants
- ...